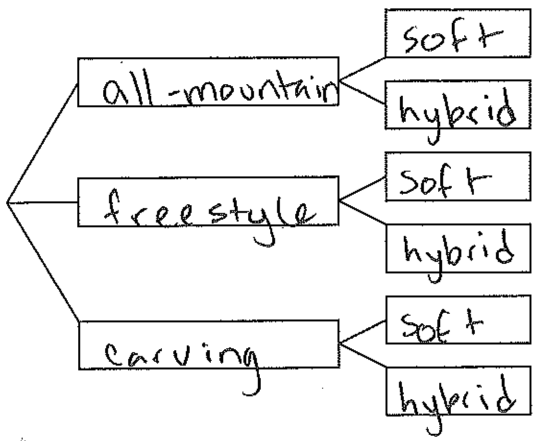
Chapter 10 – Counting Methods and Probability

Section 10.1 – Apply the Counting Principle and Permutations

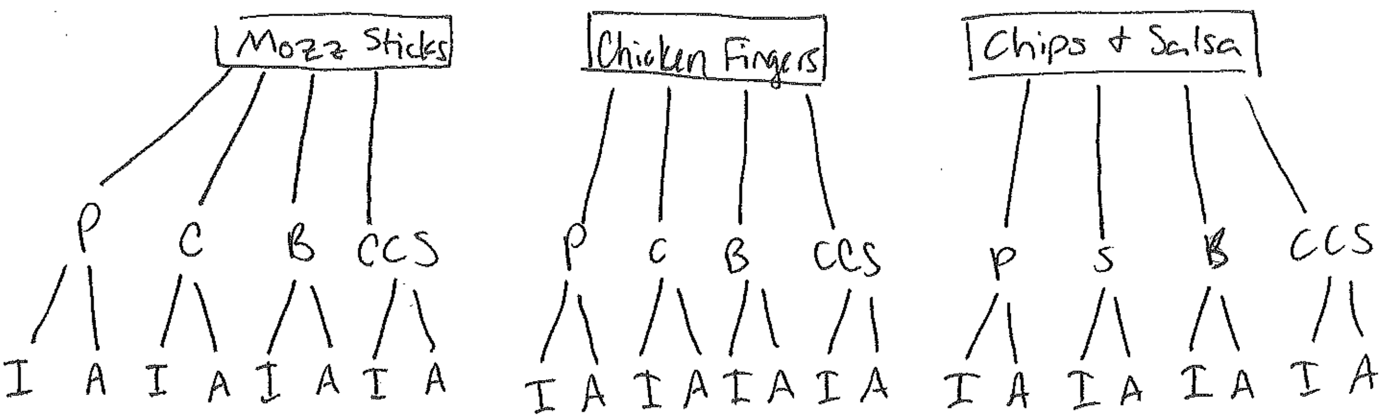
Example 1: A sporting goods store offers 3 types of snowboards (all-mountain, freestyle, and carving) and 2 types of boots (soft and hybrid). How many choices does the store offer for snowboarding equipment?

Solution: *Draw a tree diagram*



6 options

Try:
Applebee's is running a special 'You Pick 3' meal deal. You have your choice of one of three appetizers (mozzarella sticks, chicken fingers, or chips and salsa), 4 entrée's (pasta, chicken, beef, or chicken caesar salad), and 2 desserts (ice cream brownie sundae or apple crisp). Make a tree diagram to determine how many meal choices you have.



24 options

How could we arrive at this same answer without having to create a tree diagram?

$3 \cdot 4 \cdot 2 = \boxed{24}$

Fundamental Counting Principle

Two events: If one event can occur in m ways and another event can occur in n ways, then the number of ways that *both* events can occur is $m \cdot n$.

Three or more events: The fundamental counting principle can be extended to three or more events. For example, if three events can occur in m, n, and p ways, then the number of ways that *all* three events can occur is $m \cdot n \cdot p$.

Example 2

At a used book sale, you are interested in 5 novels, 3 books of nonfiction, and 7 comic books. If you buy one of each kind, how many different choices do you have?

$5 \cdot 3 \cdot 7 =$ 105 choices

Example 3

The digits 0, 1, 2, 3, and 4 are used to generate 4-digit customer codes. How many different codes are possible if digits

a. can be repeated?

$5 \cdot 5 \cdot 5 \cdot 5$

$\boxed{625}$

b. cannot be repeated?

$5 \cdot 4 \cdot 3 \cdot 2$

$\boxed{120}$

An ordering of n objects is a **permutation** of the objects. For instance, there are 6 permutations of the letters A, B, and C:

ABC **ACB** **BAC** **BCA** **CAB** **CBA**

How could we arrive at the number 6 without having to generate a list of the possible orders?

3 letters

$3 \cdot 2 \cdot 1 = 6$

3 choices for first letter

2 choices for 2nd letter

1 choice for 3rd letter

n! is read as '**n factorial**' and is equivalent to $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$. The number of permutations of n objects is n!.

Calculator $\boxed{?!\blacktriangleright}$ $\longrightarrow$ allows me to find !

Example 4
Charlie Gibson has 8 news stories to present on 'ABC World News.'

a. How many different ways can the stories be presented?

$$8! = \boxed{40,320}$$

b. If the news has to be shortened because the football game ran into overtime and only 3 of the stories can be presented, how many possible ways can a lead story, a second story, and a closing story be presented?

$$\underbrace{\underline{8} \cdot \underline{7} \cdot \underline{6}}_{\text{3 stories}} = \boxed{336}$$

The answer in part b is called a 'permutation of 8 objects taken 3 at a time' and is denoted ${}_8P_3$. We can write a special formula to evaluate this expression.

Permutations of *n* Objects Taken *r* at a Time
The number of permutations of *r* objects taken from a group of *n* distinct objects is denoted ${}_nP_r$ and is given by this formula:

$$\underset{\text{\# of objects}}{{}_n} P_{\underset{\text{how many at a time}}{r}} = \frac{n!}{(n-r)!}$$

$$\frac{8!}{(8-3)!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$

Example 5
How many different ways can 4 raffle tickets be selected from 50 tickets if each ticket wins a different prize?

r (4), n (50)

$${}_{50}P_4 = \frac{50!}{(50-4)!} = \frac{50 \cdot 49 \cdot 48 \cdot 47 \cdot \cancel{46!}}{\cancel{46!}}$$

$$= 5,527,200$$

Try:
You have 6 homework assignments to complete over the weekend. However, you only have time to complete 4 of them on Saturday. In how many orders can you complete 4 of the assignments?

$${}_6P_4 = \frac{6!}{(6-4)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = \boxed{360}$$

calculator check: Menu, prob, permutation

$nPr(6,4) =$

Permutations with Repetition – Consider the letters **S**, O, S. If you consider **S** and S to be distinct, then there are 6 permutations,

SOS **S**SO OS**S** ~~O**S**S~~ ~~S**O**S~~ ~~S**S**O~~

But, if the two occurrences of S are considered to be interchangeable, then there are only 3 distinguishable permutations:

Each one of these crossed out permutations corresponds to one of the original permutations, b/c there are 2! permutations of **S** and S.

How could we arrive at this answer without having to generate all of the permutations?

so, $\frac{3!}{2!} = \frac{3\cdot\cancel{2}\cdot\cancel{1}}{\cancel{2}\cdot\cancel{1}} = \boxed{3}$

Permutations with Repetition

The number of distinguishable permutations of n objects where one object is repeated s_1 times, another is repeated s_2 times, and so on is:

$$\frac{n!}{s_1!\,s_2!\cdots s_k!}$$

Example 6

Find the number of distinguishable permutations of the letters in

a. SOCCER

$\frac{6!}{2!} = \frac{6\cdot5\cdot4\cdot3\cdot\cancel{2}\cdot\cancel{1}}{\cancel{2}\cdot\cancel{1}}$

$= \boxed{360}$

b. SWIMMING

$\frac{8!}{\underbrace{2!}_{2\cdot1}\,\underbrace{2!}_{2\cdot1}} = \frac{8!}{4}$

$= \frac{40320}{4}$

$= \boxed{10{,}080}$

Try:

Find the number of distinguishable permutations of the letters in

TOMORROW

$\frac{8!}{3!2!} = \frac{8\cdot7\cdot6\cdot5\cdot4\cdot\cancel{3\cdot2\cdot1}}{\cancel{3\cdot2\cdot1}\cdot2\cdot1} = \frac{6720}{2}$

$= \boxed{3360}$