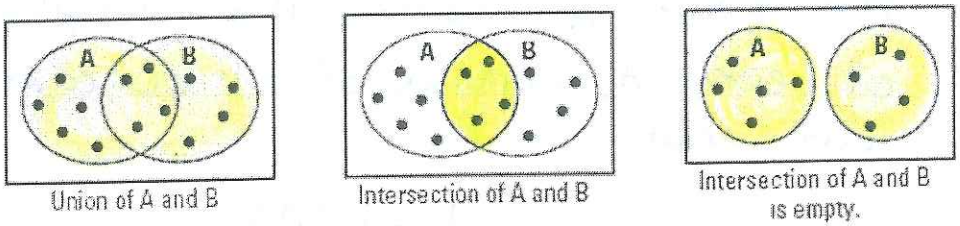
Section 10.4 – Find Probabilities of Disjoint and Overlapping Events

Union of Events A and B: consider all the outcomes for either of two events A and B

Intersection of Events A and B: consider only outcomes shared by both A and B

Compound Event: the union or intersection of two events

I. II. III.



Union of A and B | Intersection of A and B | Intersection of A and B is empty.

Overlapping Events: two events that have one or more outcomes in common.

Disjoint or Mutually Exclusive Events: two events that have no outcomes in common. (third diagram)

Probability of Compound Events

If A and B are any two events, then the probability of A or B is:

P(A or B) = $P(A) + P(B) - P(A \text{ and } B)$

If A and B are disjoint events, then the probability of A or B is:

P(A or B) = $P(A) + P(B)$

Example 1

A six-sided die is rolled. What is the probability that the number rolled is less than 3 or greater than 5?

$\frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \boxed{\frac{1}{2}} = 50\%$.

↑ less than 3 ↑ greater than 5

Example 2

A card is randomly selected from a standard deck of 52 cards. Find the probability of the given event.

10♥ 10♠ 10♦ other diamonds

a. Selecting an ace or an eight.

$\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \boxed{\frac{2}{13}}$

b. Selecting a 10 or a diamond.

$P(A) + P(B) - P(A \text{ and } B)$

$\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$

Example 3

Of 100 students surveyed, 92 own either a car or a computer. Also, 65 own cars and 82 own computers. What is the probability that a randomly selected student owns both a car and a computer?

$P(A \text{ or } B) = \frac{92}{100}$

$P(A) = \frac{65}{100}$

$P(B) = \frac{82}{100}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\frac{92}{100} = \frac{65}{100} + \frac{82}{100} - P(A \text{ and } B)$

$\frac{92}{100} = \frac{147}{100} - P(A \text{ and } B)$

$P(A \text{ and } B) = \frac{55}{100} = .55$ | 55%

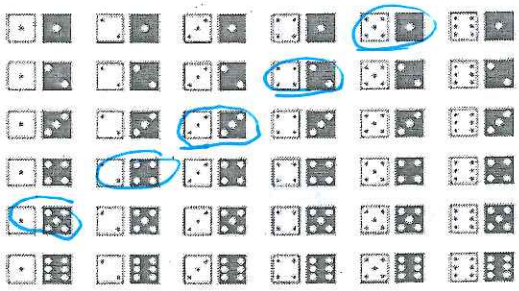
Complementary Events: The event $\bar{A}$, called the complement of event A, consists of all outcomes that are not in A.

Probability of the Complement of an Event

The probability of the complement of A is $P(\bar{A}) = 1 - P(A)$.

↑ "A Bar"

Example 4



When two six-sided dice are rolled, there are 36 possible outcomes, as shown. Find the probability of the given even

a. The sum is not 6.

$P(\text{sum is not } 6) = 1 - P(\text{sum is } 6) = 1 - \frac{5}{36} = \frac{31}{36} \approx .8611$

b. The sum is less than or equal to 9.

b) $P(\text{sum} \leq 9) = 1 - P(\text{sum} > 9) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6} = .833$

Example 5

A restaurant gives a free fortune cookie to every guest. The restaurant claims there are ~~100~~ 500 different messages hidden inside the fortune cookies. What is the probability that a group of 5 people receive at least 2 fortune cookies with the same message inside?

$P(\text{at least two are the same}) = 1 - P(\text{none are the same})$

$= 1 - \frac{500 \cdot 499 \cdot 498 \cdot 497 \cdot 496}{500^5}$

$= .0199$

# of ways $= 500^5$

# of ways to get different messages to 5 people $= 500 \cdot 499 \cdot 498 \cdot 497 \cdot 496$