

**Section 10.5 – Find Probabilities of Independent and Dependent Events**

Independent Events

Two events where the occurrence of one event has no effect on the occurrence of the other.

**Probability of Independent Events**

If A and B are independent events, then the probability that both A and B occur is:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

More generally, the probability that  $n$  independent events occur is the product of the  $n$  probabilities of the individual events.

**Example 1**

In a survey at a football game, 50 of 75 male fans and 40 of 50 female fans said that they favor the new team mascot. If 1 male and 1 female fan are randomly selected, what is the probability that both favor the new mascot?

$$\begin{aligned}
 & \begin{matrix} \swarrow \text{male} \\ P(A) \end{matrix} \cdot \begin{matrix} \swarrow \text{and} \\ P(B) \end{matrix} \begin{matrix} \swarrow \text{female} \\ \end{matrix} \\
 & \frac{50}{75} \cdot \frac{40}{50} = \frac{2000}{3750} = .5333 \\
 & = 53.33\%
 \end{aligned}$$

**Example 2**

A survey found that 46% of parents surveyed say that they read to their children at least once a week. If 3 parents are selected at random, what is the probability that all 3 will say that they read to their children at least once a week?

$$\begin{aligned}
 & \begin{matrix} P(A) & P(B) & P(C) \\ (.46) & (.46) & (.46) \end{matrix} = .0973 \\
 & = \boxed{9.73\%}
 \end{aligned}$$

**Example 3**

During each of the 5 days of a special week, an employee is randomly given 1 of 10 prizes. All prizes are available each day, and one of the prizes is a \$500 gift certificate. What is the probability that an employee receives the \$500 prize at least once?

$$\begin{aligned}
 P(\text{receives}) &= 1 - P(\text{no one gets prize}) \\
 &= 1 - \left(\frac{9}{10}\right)^5 = .4095 \\
 &= \boxed{40.95\%}
 \end{aligned}
 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} P(\text{not winning the prize on one day}) = \frac{{}^9C_1}{{}^{10}C_1} = \frac{9}{10}$$

Dependent Events

When the occurrence of one event depends on the occurrence of the other

Conditional Probability

probability that something will occur because something else occurred.  $P(B|A)$

independent →

### Probability of Dependent Events

If A and B are dependent events, then the probability that both A and B occur is:

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

← prob. that B happens given that A already happened

#### Example 4

The table shows the status of 200 registered college students. What is the probability that a randomly selected student

a. is female?

$$\frac{120}{200} = 60\%$$

b. if female, is a full time student?

$$P(F \text{ and } FT) = P(F) \cdot P(FT|F)$$

female →  $\frac{120}{200} \cdot \frac{40}{60}$  ← full time

$$= \frac{120}{200} \cdot \frac{40}{60} = 40\%$$

$$\frac{40}{120} = \frac{1}{3}$$

	Part Time	Full Time
Female	80	40
Male	60	20

120 = 80 = 200 total students  
140 60

#### Example 5

You randomly select two cards from a standard deck of 52 cards. What is the probability that the first card is a heart and the second is a club if

a. you replace the first card before selecting the second card.

← indep.

$$P(A \text{ and } B) = \frac{13}{52} \cdot \frac{13}{52}$$

$$= 6.25\%$$

b. you do not replace the first card?

← dep.

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

$$= \frac{13}{52} \cdot \frac{13}{51}$$

← after a heart is drawn

$$= 6.37\%$$

#### Example 6

You and two friends go to the same store at different times to buy costumes for a costume party. There are 15 different costumes at the store, and the store has at least 3 duplicates of each costume. What is the probability that you choose different costumes?

$$P(A \text{ and } B \text{ and } C)$$

$$= P(A) \cdot P(B|A) \cdot P(C|A \text{ and } B)$$

$$= \frac{15}{15} \cdot \frac{14}{15} \cdot \frac{13}{15}$$

$$= 80.89\%$$

← means all 3 could pick same costume

**Example 7**

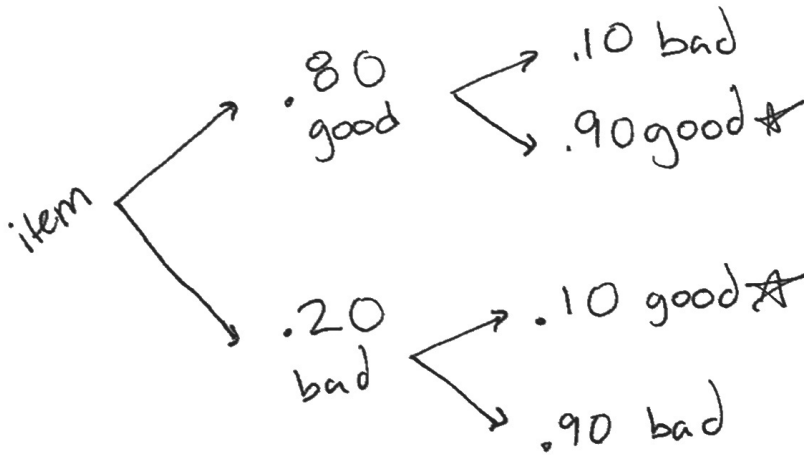
Suppose your area has 8 different Internet Service Providers (ISPs) and you and 3 friends randomly select your own ISP. What is the probability that you all choose different ISPs?

$$\frac{8}{8} \cdot \frac{7}{8} \cdot \frac{6}{8} \cdot \frac{5}{8} = \frac{105}{256}$$

$$\boxed{41\%}$$

**Example 8**

On a manufacturing line, 20% of all the items produced are defective. Although all the items are inspected before they are shipped, 10% of the items are incorrectly classified as either defective or not defective. What percent of the items will be classified as not defective?



$$P(A) \cdot P(a|A)$$

$$(.8)(.9)$$

$$.72$$

ref.

$$P(B) \cdot P(a|B)$$

$$(.2) \cdot (.1)$$

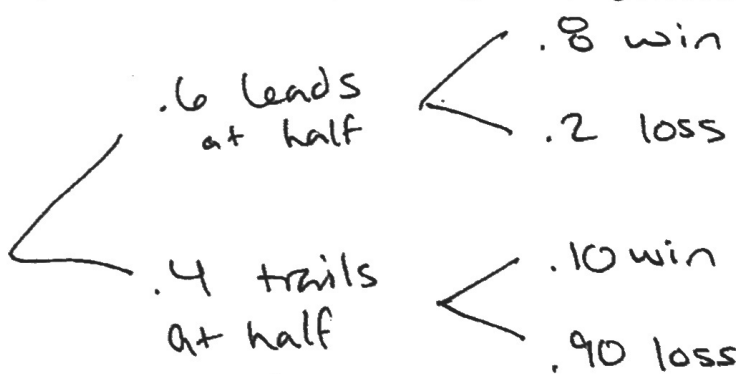
$$.02$$

$$\boxed{74\%}$$

$$.72 + .02 = .74$$

**Example 9**

A high school basketball team leads at halftime in 60% of the games in a season. The team wins 80% of the time when they have the halftime lead, but only 10% of the time when they do not. What is the probability that the team wins a particular game during the season?



$$P(\text{lead}) \cdot P(\text{win}|\text{lead})$$

$$.6 \cdot .8$$

$$.48$$

$$P(\text{trail}) \cdot P(\text{win}|\text{trail})$$

$$.4 \cdot .1$$

$$.04$$

$$.48 + .04$$

$$\boxed{52\%}$$