1st Hour

Section 10.5 – Find Probabilities of Independent and Dependent Events

Independent Events: Two events where the occurance of one event has no effect on the occurence of the other.

Probability of Independent Events

If A and B are independent events, then the probability that both A and B occur is:

P(A and B) = $P(A) \cdot P(B)$

More generally, the probability that *n* independent events occur is the product of the *n* probabilities of the individual events.

Example 1

In a survey at a football game, 50 of 75 male fans and 40 of 50 female fans said that they favor the new team mascot. If 1 male and 1 female fan are randomly selected, what is the probability that both favor the new mascot?

male, and, female

$$P(A) \cdot P(B)$$

$$\frac{50}{75} \cdot \frac{40}{50} = \frac{2000}{3750} = .5333$$

$$= 53.33\%$$

Example 2

A survey found that 46% of parents surveyed say that they read to their children at least once a week. If 3 parents are selected at random, what is the probability that all 3 will say that they read to their children at least once a week?

P(A) P(B) P(C)

$$(.46)(.46)(.46) = .0973$$

$$= \boxed{9.73\%}$$

Example 3

independent

During each of the 5 days of a special week, an employee is randomly given 1 of 10 prizes. All prizes are available each day, and one of the prizes is a \$500 gift certificate. What is the probability that an employee receives the \$500 prize at least once?

$$P(\text{recieves}) = 1 - P(\text{no one gets prize})$$

$$= 1 - \left(\tfrac{9}{10}\right)^5 = .4095$$

$$= \boxed{40.95\%}$$

$$P(\text{not winning the prize on one day}) = \frac{{}_9C_1}{{}_{10}C_1} = \boxed{\frac{9}{10}}$$

Dependent Events: when the occurence of one event depends on the occurence of the other

Conditional Probability: probability that something will occur because something else occured.

$$P(B \mid A)$$

Probability of Dependent Events

If A and B are dependent events, then the probability that both A and B occur is:

P (A and B) = $P(A) \cdot P(B \mid A)$ ← prob. that B happens given that A already happened

Example 4

The table shows the status of 200 registered college students. What is the probability that a randomly selected student

a. is female?

$$\frac{120}{200} = \boxed{60\%}$$

b. if female, is a full time student?

$$\cancel{P(F \text{ and } FT) = P(F) \cdot P(FT \mid F)}$$

$$\cancel{P(F) \cdot P(FT \mid F)}$$

female → $\cancel{\frac{120}{200} \cdot \frac{40}{60}}$ ← full time

$$\cancel{= \boxed{40\%}}$$

$$\frac{40}{120} = \boxed{\frac{1}{3}}$$

	Part Time	Full Time	
Female	80	40	= 120
Male	60	20	= 80
	140	60	200 total students

Example 5

You randomly select two cards from a standard deck of 52 cards. What is the probability that the first card is a heart and the second is a club if

a. you replace the first card before selecting the second card. ← indep.

$$P(A \text{ and } B) = \frac{13}{52} \cdot \frac{13}{52}$$

$$= 6.25\%$$

b. you do not replace the first card? ← dep.

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$= \frac{13}{52} \cdot \frac{13}{51}$$ ← after a heart is drawn

$$= 6.37\%$$

Example 6

You and two friends go to the same store at $\boxed{\text{different times}}$ to buy costumes for a costume party. There are 15 different costumes at the store, and the store has at least 3 duplicates of each costume. What is the probability that you choose different costumes?

← means all 3 could pick same costume

$$P(A \text{ and } B \text{ and } C)$$

$$= P(A) \cdot P(B \mid A) \cdot P(C \mid A \text{ and } B)$$

$$= \frac{15}{15} \cdot \frac{14}{15} \cdot \frac{13}{15}$$

$$= \boxed{80.89\%}$$

Example 7
Suppose your area has 8 different Internet Service Providers (ISPs) and you and 3 friends randomly select your own ISP. What is the probability that you all choose different ISPs?

$$\frac{8}{8} \cdot \frac{7}{8} \cdot \frac{6}{8} \cdot \frac{5}{8} = \frac{105}{256}$$

$\boxed{41\%}$

Example 8
On a manufacturing line, 20% of all the items produced are defective. Although all the items are inspected before they are shipped, 10% of the items are incorrectly classified as either defective or not defective. What percent of the items will be classified as not defective?

item → .80 good → .10 bad; .90 good ☆
item → .20 bad → .10 good ☆; .90 bad

$P(G) \cdot P(G|G)$
$(.8)(.9)$
$.72$

$P(B) \cdot P(G|B)$
$(.2) \cdot (.1)$
$.02$

$\boxed{74\%}$

$.72 + .02 = .74$

Example 9
A high school basketball team leads at halftime in 60% of the games in a season. The team wins 80% of the time when they have the halftime lead, but only 10% of the time when they do not. What is the probability that the team wins a particular game during the season?

.6 leads at half → .8 win; .2 loss
.4 trails at half → .10 win; .90 loss

$P(\text{lead}) \cdot P(\text{win}|\text{lead})$
$.6 \cdot .8$
$.48$

$P(\text{trail}) \cdot P(\text{win}|\text{trail})$
$.4 \cdot .1$
$.04$

$.48 + .04$
$\boxed{52\%}$