Section 4 – Evaluate Logarithms and Graph Logarithmic Functions

Definition of Logarithm with Base *b*

Let b and y be positive numbers with $b \neq 1$. The logarithm of y with base *b* is denoted by $\log_b y$ and is defined as follows:

$\log_b y = x$ if and only if $b^x = y$

Logarithmic Form — Exponential Form

The expression $\log_b y$ is read as " log base b of y ."

Example 1

Rewrite the following equations in logarithmic form.

a.) $2^3 = 8$ b.) $9^{1/2} = 3$ c.) $5^{-3} = \frac{1}{125}$

$\log_2 8 = 3$ $\log_9 3 = \frac{1}{2}$ $\log_5 \left(\frac{1}{125}\right) = -3$

$\log_{\text{base}}$ answer = exponent

Example 2

Rewrite the following equations in exponential form.

a.) $\log_2 32 = 5$ b.) $\log_{10} 1 = 0$ c.) $\log_{1/5} 25 = -2$ d.) $\log_5 5 = 1$

$2^5 = 32$ $10^0 = 1$ $\left(\frac{1}{5}\right)^{-2} = 25$ $5^1 = 5$

A couple of special properties:

Logarithm of 1

$\log_b 1 = 0$

b/c $b^0 = 1$

ex: $\log_5 1 = 0$ b/c $5^0 = 1$

Logarithm of *b* with Base *b*

$\log_b b = 1$

b/c $b^1 = b$

ex: $\log_5 5 = 1$

$5^1 = 5$

Example 3

Evaluate the logarithm.

a.) $\log_4 64 = \boxed{3}$ b.) $\log_3 81 = \boxed{4}$ c.) $\log_{1/4} 256 = \boxed{-4}$

$4^x = 64$ $3^x = 81$ $\left(\frac{1}{4}\right)^x = 256$

d.) $\log_{10} 0.001$ $\boxed{-3}$ e.) $\log_{64} 2 = \boxed{\frac{1}{6}}$ f.) $\log_{36} 6 = \boxed{1/2}$

$10^x = .001$ $64^x = 2$ $36^x = 6$

Special Logarithms

Common Logarithm

$\log_{10} x = \boxed{\log x}$

Natural Logarithm

$\log_e x = \boxed{\ln x}$

Example 4

Use a calculator to evaluate the logarithm.

a.) log 0.85 b.) ln 22

$10^x = .85$ $e^x = 22$

$x = \boxed{-.07}$ $x = \boxed{3.09}$

Example 5
The sales of a certain video game can be modeled by $y = 20 \ln(x - 1) + 35$ where y is the monthly number (in thousands) of games sold during the xth month after the game is released for sale ($x > 1$). Estimate the number of video games sold during the 10^{th} month after the game is released.

$y = 20 \ln(10-1)+35$

$y = \boxed{78.9 \text{ thousand}}$

<u>Inverse Functions</u>
By the definition of logarithm, it follows that the logarithmic function $g(x) = \log_b x$ is the inverse of the exponential function $f(x) = b^x$. This means that :

$g(f(x)) =$ ___ x ___ and $f(g(x)) =$ ___ x ___

$\log_b x \longrightarrow$ inverse $\longrightarrow b^x$

Example 6
Simplify the expression.

a.) $e^{\ln 9}$ — 9

b.) $10^{\log 4}$ — 4

c.) $\log_5 25^x$

$\log_5 (5^2)^x$

$\log_5 5^{2x}$

$2x$

d.) $\log_3 27^x$

$\log_3 (3^3)^x$

$\log_3 3^{3x}$

$3x$

Example 7
Find the inverse of the function.

a.) $y = 8^x$ — $y = \log_8 x$

b.) $y = 6^x$ — $y = \log_6 x$

c.) $y = \ln(x - 4)$

$x = \ln(y-4)$

$e^x = y-4$

$y = e^x + 4$

d.) $y = \ln(x + 3)$

$x = \ln(y+3)$

$e^x = y+3$

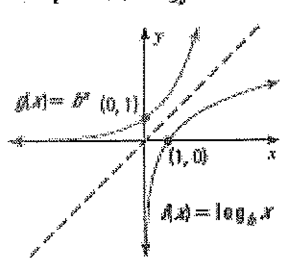
$y = e^x - 3$

Graphing Logarithmic Functions
You can use the inverse relationship between exponential and logarithmic functions to graph logarithmic functions.

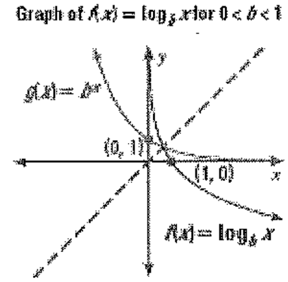
Parent Graphs for Logarithmic Functions

The graph of $f(x) = \log_b x$ is shown below for $b > 1$ and for $0 < b < 1$. Because $f(x) = \log_b x$ and $g(x) = b^x$ are inverse functions, the graph of $f(x) = \log_b x$ is the reflection of the graph of $g(x) = b^x$ in the line $y = x$.

Graph of $f(x) = \log_b x$ for $b > 1$



Graph of $f(x) = \log_b x$ for $0 < b < 1$



Note that the y-axis is a vertical asymptote of the graph of $f(x) = \log_b x$. The domain of $f(x) = \log_b x$ is $x > 0$, and the range is all real numbers.

Example 8

Graph the logarithmic functions. State the domain and range.

a.) $y = \log_3 x$

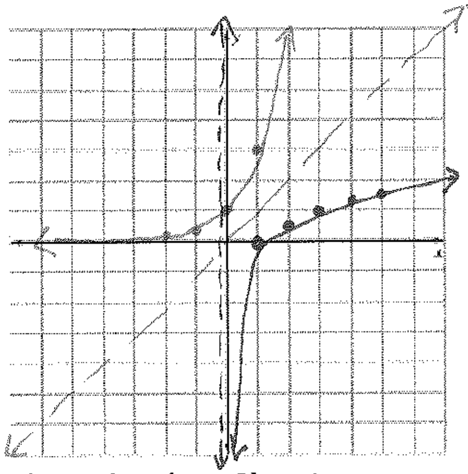
x	y
1	0
2	0.63
3	1
4	1.26
5	1.46

$y = 3^x$

D: $(0, \infty)$

R: $(-\infty, \infty)$

$3^y = x$



*notice how the asymptote also changes

b.) $y = \log_{1/2} x$

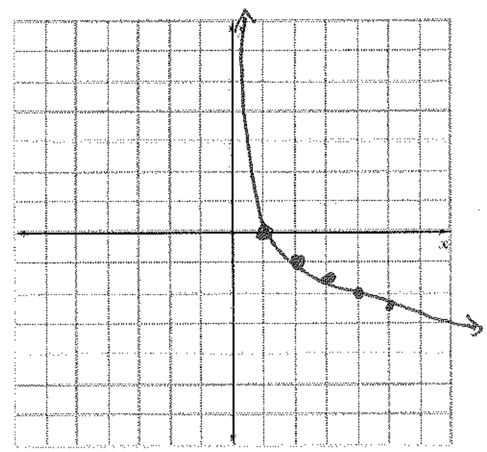
x	y
1	0
2	-1
3	-1.58
4	-2
5	-2.32

D: $(0, \infty)$

R: $(-\infty, \infty)$

$\left(\frac{1}{2}\right)^y = x$

$\frac{1}{2}^0 = 1$

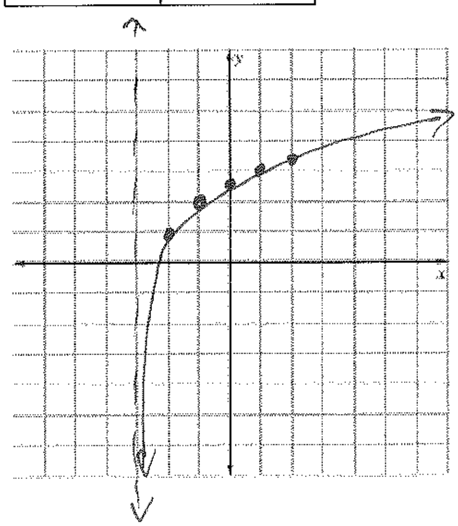


c.) $y = \log_2(x + 3) + 1$

x	y
-2	1
-1	2
0	2.58
1	3
2	3.32

D: $(-3, \infty)$

R: $(-\infty, \infty)$



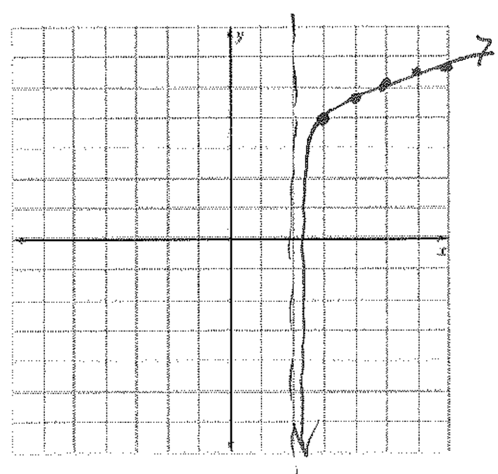
$x = -3$

d.) $y = \log_3(x - 2) + 4$

x	y
3	4
4	4.63
5	5
6	5.26
7	5.46

D: $(2, \infty)$

R: $(-\infty, \infty)$



$x = 2$