Section 5 – Apply Properties of Logarithms

Properties of Logarithms

Let b, m, and n be positive numbers such that $b \neq 1$. why? think about exponents

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

Example 1

a.) Use $\log_3 12 \approx 2.262$ and $\log_3 2 \approx 0.631$ to evaluate the logarithm.

i.) $\log_3 6$

$\log_3 \frac{12}{2} =$

$\log_3 12 - \log_3 2$

$2.262 - 0.631$

$\boxed{1.631}$

ii.) $\log_3 24$

$\log_3 (12 \cdot 2)$

$\log_3 (12) + \log_3 (2)$

$2.262 + 0.631$

$\boxed{2.893}$

iii.) $\log_3 32$

$= \log_3 (2^5)$ we know $\log_3 2$

$= 5 \cdot \log_3 (2)$

$= 5 \cdot 0.631$

$= \boxed{3.155}$

b.) Use $\log_6 5 \approx 0.898$ and $\log_6 8 \approx 1.161$ to evaluate the logarithm.

i.) $\log_6 \frac{5}{8}$

$\log_6 5 - \log_6 8$

$.898 - 1.161$

$\boxed{-.263}$

ii.) $\log_6 40$

$\log_6 (5 \cdot 8)$

$\log_6 5 + \log_6 8$

$.898 + 1.161$

$\boxed{2.059}$

iii.) $\log_6 125$

$\log_6 (5^3)$

$3 \cdot \log_6 (5)$

$3 \cdot .898$

$\boxed{2.694}$

Example 2

a.) Expand $\log_6 \frac{5x^3}{y}$

$\log_6 5x^3 - \log_6 y$

$\log_6 5 + \log_6 x^3 - \log_6 y$

$\boxed{\log_6 5 + 3\log_6 x - \log_6 y}$

b.) Expand $\log 3x^4$

$\log 3 + \log x^4$

$\boxed{\log 3 + 4 \log x}$

c.) Condense $\ln 4 + 3\ln 3 - \ln 12$

$\ln 4 + \ln 3^3 - \ln 12$

$\ln 4 + \ln 27 - \ln 12$

$\ln(4 \cdot 27) - \ln 12$

$\ln(108) - \ln 12$

$\ln \frac{108}{12} = \boxed{\ln 9}$

d.) Condense $\log 9 + 3\log 2 - \log 3$

$\log 9 + \log 2^3 - \log 3$

$\log 9 + \log 8 - \log 3$

$\log(9 \cdot 8) - \log 3$

$\log 72 - \log 3$

$\log \frac{72}{3} = \boxed{\log 24}$

Change-of-Base Formula

If a, b, and c are positive numbers with $b \neq 1$ and $c \neq 1$, then $\log_c a = \frac{\log_b a}{\log_b c}$.

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$

* allows you to evaluate any log w/ a calculator *

Example 3

Use the change-of-base formula to evaluate the logarithm.

a.) $\log_5 8$

$\frac{\log 8}{\log 5} \approx 1.292$

b.) $\log_{26} 9$

$\frac{\log 9}{\log 26} \approx 0.674$

c.) $\log_{12} 30$

$\frac{\log 30}{\log 12} \approx 1.369$

Example 4

For a sound with intensity I (in watts per square meter), the loudness L(I) of the sound (in decibels) is given by the function $L(I) = 10\log\frac{I}{I_0}$ where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). An artist in a recording studio turns up the volume of a track so that the sounds intensity doubles. By how many decibels does the loudness increase?

$L(I) = 10\log\frac{I}{I_0}$

Increase $= L(2I) - L(I)$

$= 10 \cdot \log\frac{2I}{I_0} - 10\log\frac{I}{I_0}$

$= 10 \cdot \log 2 \cdot 10\log\left(\frac{I}{I_0}\right) - 10\log\left(\frac{I}{I_0}\right)$

$= 10\log 2$

$= \boxed{3.01}$ decibels

$\log_b a = x$

$\log_b c = y$

$b^x = a$

$b^y = c$

$b^x = a$

$b^y = c$